# REVIEWS

**The Non-linear Field Theories of Mechanics** (Vol. III, Part 3, of *Encyclopaedia of Physics*). By C. TRUESDELL and W. NOLL. Springer, 1965. 602 pp. DM. 198.

Readers of an earlier review (J. W. Miles, 1963, *J. Fluid Mech.* **16**, 313) of the companion volume *Principles of Classical Mechanics and Field Theory* will know what to expect of any just review of a work written (in whole or in part) by Clifford Truesdell. I can only echo John Miles and remark with him: 'A truly definitive review of this impressive work would have been beyond my competence.... we have here a classic in the first sense of the word, in scholarship, in outlook and in command of the English language.' It is remarkable that only three years should pass before one is able to repeat his comments.

The text opens with a short introductory chapter, part philosophical, part historical. Severe treatment is accorded to those whose misconceptions of the role and significance of continuum mechanics offend the authors. Some of this verbal demolition is necessary, but some is unworthy: is it presumptuous to suggest that the authors' own work should be a sufficient justification of their point of view? It must be accepted that their rigorous methods are beyond the competence of many successful and valuable workers in applied mechanics. What we need at this juncture is a sympathetic attempt to translate into easier terms the profound concepts and powerful results of the rigorous theory. Curiously, the best so far to my mind is provided by Truesdell himself in the 1960 Socony Mobil Series of Colloquium Lectures (copies of these are available on personal request from the Socony Mobil Oil Co., Field Research Laboratory, Dallas, Texas, and it is to be hoped that their generous policy of issuing these printed versions of the now celebrated Series will continue), though other monographs attempting this task have appeared, including those by Eringen, Fredrickson, Lodge and Sedov.

The second chapter, on tensor functions, is very brief and rather forbidding, not least because of the rapid introduction of elaborate and unusual notation. For example, the tensor (dyadic) product $\mathbf{u} \otimes \mathbf{v}$ is defined in small print on p. 15 and then reappears without any further introduction in a definition on p. 23. Many readers familiar with dyadic products in the form $\mathbf{uv}$, though not necessarily with the symbolism of modern algebra, may be confused at this point.

Chapter C, on the general theory of material behaviour, and subdivided into basic principles, kinematics, the general constitutive equation, special classes of materials and fading memory, is in many ways the core of the book. It is the ideas presented at this stage that are essential to any understanding of the later chapters. The three governing principles of mechanical behaviour, those of *material frame-indifference*, of *determinism* and of *local action* are explained and their consequences developed. Naturally the chapter leans heavily on the work of Coleman and Noll, though it is broadly expository and makes full mention of other approaches. Distinction is drawn between simple materials (whose constitutive equations involve functionals of only the first spatial derivatives of the velocity vector) and those of higher grade (where higher spatial derivatives also are involved); in a continuum theory, materials are simple by assumption, not by deduction. Further division into simple fluids and simple solids follows, though the existence of other classes is illustrated by reference to Wang's work on simple subfluids. Definition of these classes of material results from their invariance properties, not from their functional dependence. Although physically rather easy to grasp, the Noll theory of materials of fading memory involves rather elaborate mathematical concepts; I experienced some difficulty in interpreting the definition of *materials of the differential type of grade n*, for I have always regarded the higher derivatives involved (i.e. the higher-order Rivlin–Ericken tensors) as time derivatives in a body co-ordinate system, rather than as the spatial derivatives referred to earlier.

Chapter D, the longest, is on elasticity including elastic, hyper-elastic and hypo-elastic materials. This however is not the journal in which to discuss this chapter, so we can pass on to the last chapter, on fluidity. Here, some contact with fluid mechanics, as ordinarily understood, is made. Particular flow patterns are considered for simple fluids, which are redefined in terms of two basic assumptions:

'(i) *The present stress is determined by the history of the gradient of the deformation function.*

(ii) *A simple fluid has the maximum possible material symmetry.*'

In particular, the significance of three scalar material functionals* for lineal and viscometric flows is emphasized; use of them effects a major simplification in most applications. Some results are given in traditional co-ordinate notation for simple shearing, channel, and general helical flows. Attention is paid to viscometry, and illustrative photographs are given. However, the author's touch is less sure here: inferences drawn from crude mathematical arguments are applied to observations in complex physical circumstances in a manner unrigorous by any standards, let alone those insisted upon earlier. For example, it is argued on pp. 455 *et seq.* that normal stresses will lead to swelling at the exit from a pipe (the so-called Merrington effect) and because such swelling is usually observed, it is then argued that the inequality

$$2\sigma_2(\gamma) - \sigma_1(\gamma) > 0$$

holds, $\sigma_1$ and $\sigma_2$ being normal stress functions of $\gamma$, the shear rate. This argument is naïve, even though the result may be correct, because we do not even know whether the problem posed in its simplest form has a unique solution. We do know however that the velocity field is dependent on the pressure field, and that it is extremely unlikely that the pressure will be a function only of distance along the tube right up to the end, as is assumed on p. 457. The chapter continues with a discussion of special simple fluids, including Truesdell's theory of 'Stokesian' fluids, which involves thermodynamic quantities such as tem-

* Sometimes called viscosity and normal stress differences.

perature, and asymptotic approximations. Other fluids, particularly Ericksen's anisotropic fluids, are dealt with at the end of the chapter.

The list of works cited runs to about 1000 references, many of them to long and involved original papers.

The ordinary reader must marvel at such erudition, but he may also reasonably wonder whether the worker in traditional fluid mechanics (i.e. hydro- or aero-dynamics) need concern himself with such an elaborate approach. The simple answer is that he need not. However, it is worth bearing in mind that the lack of a satisfactory rheological theory of general materials has until recent years seriously hampered progress in understanding the behaviour of 'gunky' rather than simple Newtonian fluids. Because their flow behaviour has not been seriously studied by applied mathematicians—there are a few honourable exceptions—there has I think been a tendency to minimize their importance. The equivalent situation no longer applies in solid mechanics—where the study of visco-elastic and plastic materials is well advanced—and I believe that fluid mechanics will in future be increasingly concerned with elastico-viscous and other such fluids. For this reason, a sympathetic study of this book, particularly of Chapter C, could be rewarding for those interested in the teaching of fluid mechanics: it might suggest orientating undergraduate teaching rather more towards advanced vector and dyadic analysis, and the study of (three-dimensional Euclidean) co-ordinate systems and their transformations, than is usual at present. From the point of view of later applications, there is even a case for putting general tensor analysis into undergraduate curricula as a necessary preliminary to mechanics!

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## Radiative Contributions to Energy and Momentum Transport in a Gas.

By D. H. Sampson. Interscience, 1965. 178 pp. £3. 4*s*.

This little book, one of the publisher's Tracts on Physics and Astronomy, treats the title subject for quite general conditions. The photon approach is used to derive the radiative contributions to the Boltzmann equation and it is shown how matter and radiation contributions combine to form the complete macroscopic transport equations. The general solution to the radiative transport equation is given and a number of important special cases involving simplifying assumptions are treated. The case when the radiating matter is not in local thermodynamic equilibrium is dealt with in some detail and the conditions required for equilibrium are discussed.

The book will be useful to all workers in the combined field of radiation and fluid mechanics. It expects the reader to have a fair background in statistical mechanics and kinetic theory. It contains a full list of references, but would probably have appealed to a wider audience and been more useful as a text for advanced courses if it had contained more specific references to the required background literature. The going is quite heavy at times, and might have been eased by the inclusion of a few more illustrative diagrams.

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